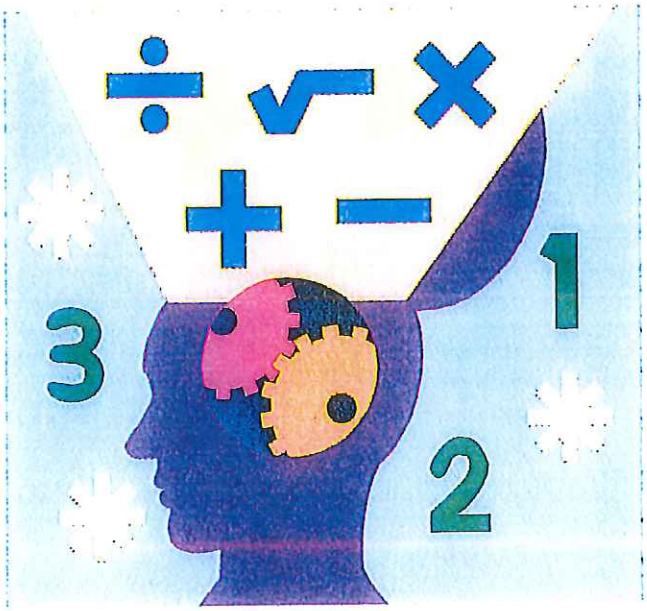
St Mary's C of E (Aided) Primary School, Pulborough

A Christ centred school with a child centred curriculum

Progression in Calculation



J.Warner November 2013

Understanding the concepts

+ Understanding Addition +

- Addition can be seen as :
  - combining two sets to make a total
  - counting on steps along a numberline
- Adding zero leaves a number unchanged
- Addition reverses subtraction
- More than two numbers can be added
- Addition can be done in any order

+ Vocabulary for Addition +

More
more than
greater than
count on
add
sum
total
altogether
plus

- Understanding Subtraction -

- Subtraction can be seen as :
  - taking away
  - finding the difference between
  - counting on, as when working out change
- Subtracting zero leaves a number unchanged
- Subtraction reverses addition
- Subtraction cannot be done in any order

- Vocabulary for Subtraction -

take away
subtract
count back
less than
fewer than
difference between
minus
decrease

× Understanding Multiplication ×

- Multiplication can be seen as:
  - repeated addition: 5 × 3 = 5 + 5+ 5
  - describing an array:
    e.g. 4 × 2 as ∘ ∘ ∘ ∘
    ∘ ∘ ∘ ∘
- Multiplication can be done in any order
- Halving reverses doubling
- Multiplication reverses division
- Multiplication by 1 leaves a number unchanged
- Multiplication by zero results in zero

× Vocabulary for Multiplication ×

double
times
multiply
multiplied by
multiple of
lots of
groups of
product
factor

÷ Understanding Division ÷

- Division can be seen as:
  - sharing equally: 18 sweets shared equally between 3 people
  - grouping (repeated subtraction): 18 sweets, how many packets of 3?
- Division cannot be done in any order
- Dividing a whole number by 1 leaves the number unchanged
- Division is the inverse of multiplication

÷ Vocabulary for Division ÷

share
group
halve
divide
divided by
equal groups
left over
remainder
divisible by
quotient

Mental Calculation Strategies

We want children to carry out calculations in their heads whenever possible. We need to give them a repertoire of strategies so that they can be efficient and accurate. Here are some examples:

+ Addition +

• Know addition facts for all numbers to 20	
• Put the larger number first	2 + 36 = 36 + 2
• Partitioning and recombining	55 + 37 = 50 + 30 + 5 + 7
• Bridging through multiples of 10	57 + 14 = 57 + 3 + 10 + 1
• Adding 9 or 11	58 + 9 = 58 + 10 - 1 58 + 11 = 58 + 10 + 1
• Using near doubles	38 + 35 is double 35 add 3

- Subtraction -

• Know subtraction facts corresponding to addition facts to 20	
• Partitioning and recombining	58 - 32 = (50 -30) + (8-2)
• Bridging through multiples of 10	57 - 12 = 57 - 7 - 5
• Subtracting 9 or 11	58 - 9 = 58 - 10 + 1 58 - 11 = 58 - 10 - 1
• Find a small difference between two numbers by counting on	

× Multiplication ×

• Know multiplication facts - all facts to 10 × 10 (and know that other facts can be derived from knowing these)	
• Multiply by multiples of 10 by moving digits to the left	h t u 2 8 × 10 2 8 0 th h t u 2 8 × 100 2 8 0 0
• Partition to multiply	47 × 5 = (40 × 5) + (7 × 5)
• Multiply by using factors	45 × 20 = 45 × 10 × 2
• Use doubling	17 × 4 = 17 × 2 × 2
• Use closely related facts	9 × 49 = (10 × 49) - 49

÷ Division ÷

• Know division facts corresponding to multiplication facts 10 × 10	
• Divide by multiples of 10 by moving digits to the right	h t u 4 8 0 ÷ 10 4 8 th h t u .10ths 100ths 7 3 6 5 ÷ 100 7 3 . 6 5
• Use halving	96 ÷ 8 = 96 ÷ 2 ÷ 2 ÷ 2
• Use knowledge of table facts and place value	560 ÷ 8 = 70 because 56 ÷ 8 = 7 therefore 560 ÷ 8 = 70

Stages in Addition

Representing and Early Recording	Jottings/Informal Methods			Formal Written Methods
Pictorial Ideas	Number Lines	Partitioning	Number Lines	Vertical Layout

Pictorial Ideas

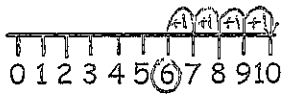
6 + 4 = 10

Counting on using pictures:

○ ○ ○ ○ ○ ○ / ○ ○ ○ ○

6 4

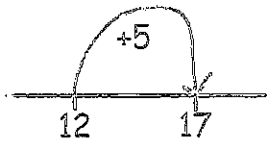
Counting on using a number line:



Number Lines

12 + 5 = 17

Empty number line



Partitioning

13 + 28 = 41

Adding tens first then units and recombining:

20 + 10 = 30
8 + 3 = 11
30 + 11 = 41

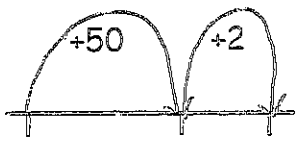
Using a number line with partitioning, add on tens then units:

+10 +3

0 28 38 41

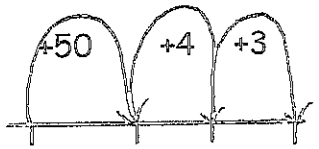
Number Lines

141 + 52 = 193



Bridging through 10:

86 + 57 = 143



Vertical Layout

1. *Introduce with expanded working, by partitioning the numbers:*

Start with 2 digit numbers:

47 + 76

40 + 7
70 + 6
110 + 13 = 123

Extend to 3 digit numbers:

368 + 493

300 + 60 + 8
400 + 90 + 3
700 + 150 + 11 = 861

2. Now use expanded working but without partitioning:

h t u	
4 7	
+ 7 6	
1 3	(7+6)
1 1 0	(40+70)
1 2 3	

h t u	
3 6 8	
+ 4 9 3	
1 1	(8 +3)
1 5 0	(60+90)
7 0 0	(300+400)
8 6 1	

- Brackets could be removed in time

3. Final stage - contract the working to a compact, efficient form:

h t u
4 7
+ 7 6
1 2 3
1 1

h t u
3 6 8
4 9 3
8 6 1
1 1

4. Extend to bigger numbers and decimals.

- Stages in Subtraction -

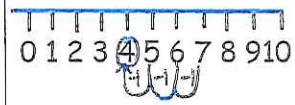
Representing and Early Recording	Jottings/Informal Methods		Formal Written Methods
Pictorial Ideas	Number Lines	Partitioning	Vertical Layout

Pictorial Ideas

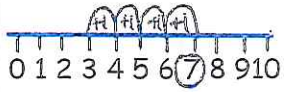
7 – 3 = 4

Counting back using pictures:

Counting back using a number line:



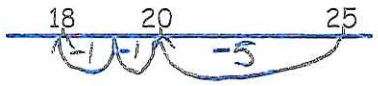
Counting on using a number line(finding the difference):



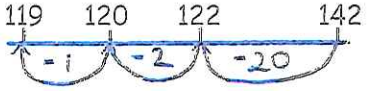
Number Lines

Take Away

25 – 7 = 18



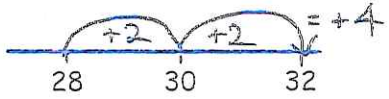
143 – 23 = 119



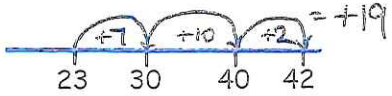
OR

Finding the Difference (by counting on)

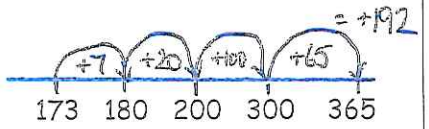
32 – 28 = 4



42 – 23 = 19



365 – 173 = 192



Partitioning

86 – 34 = 52

Partition into tens and units:

80 – 30 = 50

6 – 4 = 2

Then recombine the tens and units:

50 + 2 = 52

Or

Partition the number that is being subtracted and take that away:

86 – 30 = 56

56 – 4 = 52

365 – 123 = 242

Partition into hundreds, tens and units:

300 – 100 = 200

60 – 20 = 40

5 – 3 = 2

Then recombine the hundreds, tens and units:

200 + 40 + 2 = 242

Or

Partition the number that is being subtracted and take that away:

365 – 100 = 265

265 – 20 = 245

245 – 3 = 243

When adjustment is needed:

324 – 69 = 255

200 ~~300~~ – 0 = 200

110 ~~20~~ – 60 = 50

14 – 9 = 5

Then recombine the hundreds, tens and units:

200 + 50 + 5 = 255

Vertical Layout

1. Introduce with expanded working, by partitioning the numbers (Decomposition):

Start with no adjustment needed:

86 – 34

80 + 6

30 + 4

50 + 2 = 52

968 – 453

900 + 60 + 8

400 + 50 + 3

500 + 10 + 5 + 515

Now use expanded working but with adjustment:

968 – 389

800 150 1

~~900~~ + ~~60~~ + 8

300 + 80 + 9

500 + 70 + 9 = 579

2. Contract the working to a compact, efficient form:

Start with no adjustment needed:

h t u

968

- 453

515

Move on to adjustments:

h t u

8 15 1

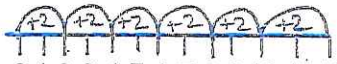
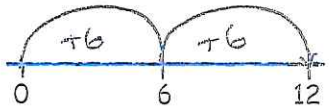
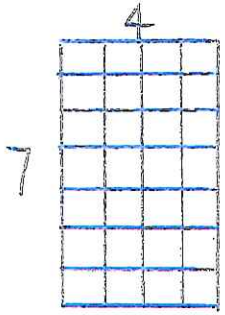
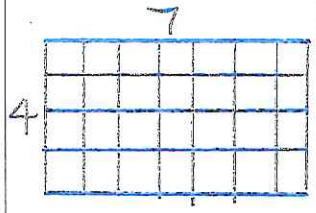
~~9~~ ~~6~~ 8

- 3 8 9

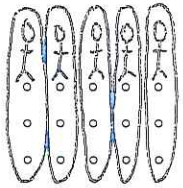
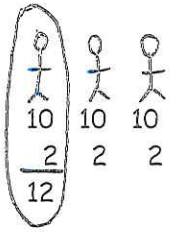
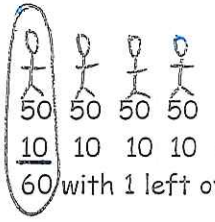
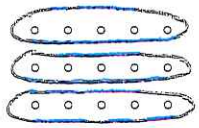
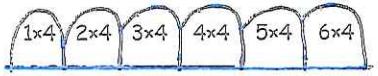
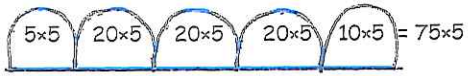
5 7 9

3. Extend to bigger numbers and decimals.

Stages in Multiplication

LOTS OF	REPEATED ADDITION	ARRAYS	GRID METHOD	COLUMN MULTIPLICATION
3 lots of 2 is 6 ∘∘∘ ∘∘∘ 2 lots of 3 is 6 ∘∘ ∘∘ ∘∘	Counting in twos.. etc. 6 ×2 = 12 2 + 2 + 2 + 2 + 2 + 2 = 12  Or 2 × 6 6 + 6 = 12  	7 × 4 = 28  4 × 7 = 28  	1. Start by multiplying 2 digit numbers by 1 digit numbers: 38 × 7 | × | 30 | 8 | | 7 | 210 + 56 | = 266 (children use knowledge that 7 × 3 = 21 and then multiply 21 by 10 to work out answer to 7 × 30) Move onto multiplying 3 digit numbers by 1 digit numbers: 425 × 6 | × | 400 | 20 | 5 | | 6 | 2400 + 120 + 30 | = 2550 (children use knowledge that 6 × 4 = 24 and then multiply 24 by 100 to work out answer to 6 × 400) 2. Extend to multiplying by 2 digit numbers: 56 × 27 | × | 50 | 6 | | 20 | 1000 + 120 | = 1120 | 7 | 350 + 42 | =+ 392 1512 1	1. Start with expanded working to reinforce understanding: 38 × 7 56 (7 × 8) 210 (7 × 30) 266 56 × 27 42 (7 × 6) 350 (7 × 50) 120 (20 × 6) 1000 (20 × 50) 1512 1 2. Once children are confident move to compact working: 38 × 7 266 5 56 × 27 392 (7 × 56) 1120 (20 × 56) 1512 1 (Once children are secure with this final stage there is no need to record the calculations at the side) 3. Extend to bigger numbers and decimals.

Stages in Division

SHARING	GROUPING	CHUNKING	BUS STOP
15 ÷ 5 = 3 Question: *5 people share 15 sweets equally. How many does each person get?*  Each child gets 3 sweets. 36 ÷ 3 = 12 Question: *A class of 36 split into 3 equal teams. How many are in each team?*  10 10 10 — 3 × 10 = 30 2 2 2 — 3 × 2 = 6 12 — 36 Each team has 12 children in it. 241 ÷ 4 = 60 r 1  50 50 50 50 — 4 × 50 = 200 10 10 10 10 — 4 × 10 = 40 60 with 1 left over — 240	15 ÷ 5 = 3 Question: *How many bags of 5 sweets can you make with 15 sweets?*  Grouping on a numberline: 24 ÷ 4 = 6 Question: *How many groups of 4 children in a class of 24?*  0 4 8 12 16 20 24 379 ÷ 5 = 75 r 4 Grouping towards the target number from zero:  0 50 100 200 300 350 375 or Grouping backwards from the target number towards zero:  4 29 129 229 329 379	The written methods used are based on the idea of repeated subtraction e.g. to solve 20 ÷ 5 we can think of it as 20 - 5 - 5 - 5 - 5, five multiplied by 4 is 20 or there are four fives in twenty. For bigger numbers we need to take off bigger chunks in order for this to be an efficient method. 72 ÷ 5 72 - 50 — 10 × 5 22 - 20 — 4 × 5 2 72 ÷ 5 = 14 remainder 2 Children then move on to whatever size chunks they feel confident with: 256 ÷ 7 256 - 140 — 20 × 7 116 - 70 — 10 × 7 46 - 42 — 6 × 7 4 256 ÷ 7 = 36 remainder 4	This final stage contracts the working to a compact, efficient form. 196 ÷ 4 = 32 r 4 32 r 4 6 ⟌ 196 - 180 — 30 × 6 16 - 12 — 2 × 6 4 Children aim to choose the largest possible multiples of 100,10 and then 1 as appropriate: 972 ÷ 36 = 27 27 36 ⟌ 972 - 720 — 20 × 36 252 - 252 — 7 × 36 0 Once confident the children no longer need to record the workings at the side: 730 ÷ 5 = 146 146 (2 3) 5 ⟌ 730 Extend to bigger numbers and decimals.

Terminology

Arrays - An arrangement of shapes, objects, marks to support a visual model of 'lots of' leading into multiplication.

Bus stop - Method of division in the final stages of development that a child may use when they have mastered previous concepts.

Chunking - Used when sharing to distribute amounts out in multiples of a chosen number.

Complimentary addition - Used as a method to help children with subtraction. Children are encouraged to add on using a number line until they reach their destination number.

Counting back - Counting underneath a number line to show subtraction.

Counting on - Adding along a number line.

Column method - Standard calculation method used when children have good mastery of stages.

Decomposition - The final stage of column subtraction used when a child has a very good sense of understanding of taking away.

End destination - Used when talking about the number children have to reach when adding on a number line for example in complimentary addition.

Extended column - Used to support learning in the final stages of addition and multiplication prior to the column method.

Finding the difference - Used in subtraction for children to calculate the difference between two numbers.

Grid Method - Method by which children are taught to multiply larger numbers.

Grouping - Important method used in division when the child is encouraged to group on a number line to find answers to problems such as: How many classes, (groups), can you have of 25 children in a school of 125 pupils?

Lots of - Used to help children think of multiplying.

Mental and oral starter - Quick fire warm up to mathematics lessons where children apply their knowledge and understanding of number.

Number line - A tool for children to use to support calculation.

Partitioning - Children are taught to see tens and units, hundreds tens and units.

Sharing - Equally distributing amounts in division problems.